Common Fixed Point Theorems in Cone Metric Spaces under G -Type Control Function

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Abstract

Fixed point theory plays an important role in functional analysis, approximation theory, differential equations and applications such as boundary value problems etc. The theory itself is a beautiful mixture of analysis, topology, and geometry. Over the last 80 years or so the theory of fixed points has been revealed as a very powerful and important tool in the study of nonlinear phenomena. In particular, fixed point techniques have been applied in such diverse fields as Biology, Chemistry, Economics, Engineering, Game Theory, and Physics. Fixed point theory broadly speaking demonstrates the existence, uniqueness and construction of fixed points of a function or a family of functions under diverse assumptions about the structure of the domain X (such as X may be a metric space or normed linear space or a topological space) of the concerned functions. In 2007, Huang & Xian [7] introduced the notion of a cone metric space and established some fixed point theorems in cone metric spaces, an ambient space which is obtained by replacing the real axis in the definition of the distance, by an ordered real Banach[3] space whose order is induced by a normal cone P. In this paper we prove the existence of coincidence points and common fixed points for large class of a almost contractions in cone metric spaces and obtain results of Berinde[4] as a corollary in G -cone metric spaces.

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1. Introduction

Fixed point theory plays an important role in functional analysis, approximation theory, differential equations and applications such as boundary value problems etc. The theory itself is a beautiful mixture of analysis, topology, and geometry. Over the last 80 years or so the theory of fixed points has been revealed as a very powerful and important tool in the study of nonlinear phenomena. In particular, fixed point techniques have been applied in such diverse fields as Biology, Chemistry, Economics, Engineering, Game Theory, and Physics. Fixed point theory broadly speaking demonstrates the existence, uniqueness and construction of fixed points of a function or a family of functions under diverse assumptions about the structure of the domain X (such as X may be a metric space or normed linear space or a topological space) of the concerned functions. The concept of a metric space was introduced in 1906 by M. Frechet [6]. It furnishes the common idealization of a large number of mathematical, physical and other scientific constructs in which the notion of a "distance" appears.

One extension of metric spaces is the so called cone metric space. In cone metric spaces, the metric is no longer a positive number but a vector, in general an element of a Banach space equipped with a cone. In 2007, Huang & Xian [7] introduced the notion of a cone metric space and established some fixed point theorems in cone metric spaces, an ambient space which is obtained by replacing the real axis in the definition of the distance, by an ordered real Banach space whose order is induced by a normal cone P as follows:

Definition 1.1 : (Huang & Xian [7]) Let E be a real Banach space and P a subset of E. P is called a cone if

(i) P is closed, non-empty and $P \neq \{ 0 \}$

(ii) $ax + by \in P$ $\forall x, y \in P$ and non-negative real numbers a and b.

(iii) $P \cap (-P) = \{0\}$.

Definition 1.2 : (L. G. Huang, Z. Xian ,[7])

We define a partial ordering $\leq$ on E with respect to P and $P \subset E$ by $x \leq y$ if and only if $y - x \in P$. We shall write $x << y$ if $y - x \in$ int P, int P denotes the interior of P. We denote by $\| . \|$ the norm on E. The cone P is called normal if there is a number K > 0 such that for all $x, y \in E$, $0 \leq x \leq y$ implies

$$\|x\| \leq K \|y\| \quad \ldots \quad (1.2.1)$$

The least positive number K satisfying (1.2.1) is called the normal constant of P.

Definition 1.3: (L. G. Huang, Z. Xian ,[7])

A cone P is called regular if every increasing sequence which is bounded from above is convergent. That is, if $\{x_n\}_{n\geq1}$ is a sequence such that $x_1 \leq x_2 \leq \ldots \leq y$ for some $y \in E$,then there is $x \in E$ such that $\lim_{n\to\infty}\|x_n - x\| = 0$

Definition 1.4 : (L. G. Huang, Z. Xian ,[7])

E is a real Banach space, P is a cone in E with $\mathrm{int}P \neq \emptyset$ and $\leq$ is the partial

ordering with respect to P. Let X be a non-empty set and $d: X \times X \to P$ a

mapping such that

(d_1) $0 \leq d(x, y)$ for all $x, y \in X$ (non - negativity)

(d_2) $d(x, y) = 0$ if and only if $x = y$.

(d_3) $d(x, y) = d(y, x)$ for all $x, y \in X$ (symmetry)

(d_4) $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$ (triangle inequality)

Then d is called a cone-metric on X and (X, d) is called a cone metric space.

Example 1.5 : (L. G. Huang, Z. Xian ,[7])

Let $E = R^2, P = \{(x, y) \in E / x, y \geq 0\}, X = R$ and $d : X \times X \rightarrow P$ defined by

$d(x, y) = (|x - y|, \alpha |x - y|)$ where $\alpha \geq 0$ is a constant.

Then (X , d) be a cone metric space.

Definition 1.6 : (L. G. Huang, Z. Xian ,[7])

Let (X, d) be a cone metric space, $x \in X$ and $\{x_n\}_{n \geq 1}$ a sequence in X. Then

(i) $\{x_n\}_{n\geq1}$ converges to x whenever for every $c \in E$ with $0 << c$ there is a natural number N such that $d(x_n, x) << c$ for all $n \geq N$.

We denote this by $\lim_{n\to\infty} x_n = x$ or $x_n \rightarrow x$ as $n \rightarrow \infty$

(ii) $\{x_n\}_{n\geq1}$ is said to be a Cauchy Sequence if for every $c \in E$ with $0 << c$ there is a natural number N such that $d(x_n, x_m) << c$ for all $n, m \geq N$.

(iii) (X, d) is called a complete cone metric space if every Cauchy Sequence in X is convergent.

L. G. Huang, Z. Xian [7] proved some fixed point theorems of contractive mappings, which generalize the existing results in metric spaces such as Banach [3], Kannan [9] etc.

In 2008, Rezapour and Hamlbarani [11] , proved that there are no normal cones with normal constant $M < 1$. Further, in [11] it is shown that for $k > 1$ there are cones with normal constant $M > k$. An example of a non normal cone is given in [11]. Further, Rezapour and Hamlbarani [11] obtained generalizations of the results of L. G. Huang, Z. Xian [7].

Definition 1.7 :

Let f and g be self maps of a non empty set X. If $\omega = fx = gx$ for some x in X, then x is called a coincidence point of f and g and ω is called a point of coincidence of f and g.

Definition 1.8 :

Let f and g be two self-maps defined on a nonempty set X. Then f and g are

said to be weakly compatible if they commute at their coincidence points.

That is , $fx = gx$ implies $gfx = fgx$

In 2008, M.Abbas and G.Jungck [1] obtained several coincidence and common

fixed point theorems for mappings defined on a cone metric space.

The following proposition is easy to prove .

We use this proposition in the main results.

Proposition 1.9: Let f and g be weakly compatible self maps of a non empty set X. If f and g have a unique point of coincidence $\omega = fx = gx$, then ω is the unique common fixed point of f and g.

In this paper we prove the existence of coincidence points and common fixed points for large class of a almost contractions in cone metric spaces and obtain results of Berinde[4] as a corollary in G -cone metric spaces.

Berinde [4] proved the following theorem for an almost contractive self map on a complete metric space.

Theorem 1.10 (V. Berinde [4], Theorem 1): Let (X, d) be a complete metric space and T : X → X an almost contraction, that is a mapping for which there exist aconstant $\mu \in (0,1)$ and some H ≥ 0 such that

$d(Tx,Ty) \leq \mu\, d(x,y) + H d(y,Tx)$, for all $x,y \in X$ … (1.10.1)

Then

1. $F(T) = \{x \in X : Tx = x\} \neq \varphi$
2. For any $x_0 \in X$, the Picard iteration $\{x_n\}_{n=0}^{\infty}$ given by $x_{n+1} = Tx_n$ converges to some $x^* \in F(T)$;
3. The following estimate holds

$d(x_{n+i-1}, x^*) \leq \frac{\mu^i}{1-\mu} d(x_n,x_{n-1})$, n = 0,1.2,…, i = 1,2

This theorem concludes that T has a fixed point. However, the fixed point need not be unique in view of the following example.

Example 1.11: Define T : [0,1] → [0,1] by $T(x) = \begin{cases} 0, & x = 0 \\ 1, & x = 1 \end{cases}$

Then T satisfies (1.10.1) and T has two fixed points.

Berinde [4] extended this result as a coincidence theorem to two self maps on a

cone metric space(X,d) as follows

Theorem 1.12 (V. Berinde [4], Theorem 2): Let (X, d) be a cone metric space and T, S : X → X be two mappings for which there exist aconstant $\mu \in (0,1)$ and some H ≥ 0 such that

$d(Tx,Ty) \leq \mu\, d(Sx,Sy) + H d(Sy,Tx)$, for all $x,y \in X$ … (1.12.1)

If the range of S contains the range of T and S(X) is a complete subspace of X, then T and S have a coincidence point in X. Moreover, for any $x_0 \in X$, the iteration $\{Sx_n\}$ defined by $Sx_{n+1} = Tx_n$ converges to some coincidence point x^* of T and S.

The coincidence point obtained from theorem (1.12) need not be unique in view of example (1.11) (by taking S = T).

In order to obtain a common fixed point theorem from the above coincidence point theorem Berinde [4] imposed an additional contractive condition which makes the coincidence point unique and hence becomes a common fixed point.

Theorem 1.13 (V. Berinde [4], Theorem 3): Let (X, d) be a cone metric space and T, S : X → X be two mappings satisfying (1.12.1) for which there exist a constant $\tau \in (0,1)$ and some M ≥ 0 such that

$d(Tx,Ty) \leq \tau\, d(Sx,Sy) + M\, d(Sx,Tx)$, for all $x,y \in X$ … (1.13.1)

If the range of S contains the range of T and S(X) is a complete subspace of X, then T and S have a unique coincidence point in X. Moreover, if T and S are weakly compatible, then T and S have a unique common fixed point in X. In both cases, for any $x_0 \in X$, the iteration $\{Sx_n\}$ defined by $Sx_{n+1} = Tx_n$ converges to unique common fixed point(coincidence point) x^* of T and S.

Babu et.al [2] unified (1.12.1) and (1.13.1) in the metric space context for a single

map and obtained the following theorem.

Theorem 1.14 (G. V. R. Babu [2], Theorem 2.3): Let (X, d) be a complete metric space and let T : X →X be a map satisfying the condition

$d(Tx,Ty) \le \tau\, d(x,y) + M \min\{d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx)\}$ for all $x,y \in X$

Then T has a unique common fixed point.

In the cone metric space context of the above theorem, Berinde [4] has proved the

following Theorem 1.15.

Theorem 1.15 (V. Berinde [4], Theorem 4): Let (X, d) be a cone metric space and T, S : X → X be two mappings satisfying (1.12.1) for which there exist a constant $\tau \in (0,1)$ and some $M \ge 0$ such that
$d(Tx,Ty) \le \tau\, d(Sx,Sy) + M \min\{d(Sx,Tx), d(Sy,Ty), d(Sx,Ty), d(Sy,Tx)\}$

for all $x,y \in X$ …(1.15.1)

If the range of S contains the range of T and S(X) is a complete subspace of X, then T and S have a unique coincidence point in X. Moreover, if T and S are weakly compatible, then T and S have a unique common fixed point in X. In both cases, for any $x_0 \in X$, the iteration $\{Sx_n\}$ defined by $Sx_{n+1} = Tx_n$ converges to unique common fixed point(coincidence point) x^* of T and S.

Note: In Theorem 1.15 the right hand side of the equation (1.15.1) may not bemeaningful, since $\min\{d(Sx,Tx), d(Sy,Ty), d(Sx,Ty), d(Sy,Tx)\}$ may not exist inP (cone of E).
In order to overcome this difficulty, in the next section we introduce the concept of G-type control function, and obtain a satisfactory account of the above theorem.

2. Main Results

In this section we introduce the concept of G-type control function and obtain a satisfactory account of theorem (1.15)

Definition 2.1: Let E be a real Banach Space and P a cone in E. Suppose $\varphi: P^4 \to P$ is a continuous function which satisfies the condition
(G) : $\varphi(p_1,p_2,p_3,p_4) = 0$ if any one of p_1,p_2,p_3,p_4 is zero.Then φ is called a G– type control function.

Example 2.2: Let E be a Banach space, P be a cone in E. Let p_1,p_2,p_3,p_4 be bounded linear functional on E. Define $\varphi: P^4 \to P$ by
$\varphi(p_1,p_2,p_3,p_4) = |f_1(p_1) f_2(p_2) f_3(p_3) f_4(p_4)|(p_1+p_2+p_3+p_4)$
Then φ is continuous and satisfies $\varphi(p_1,p_2,p_3,p_4) = 0$ if any one of p_1,p_2,p_3,p_4 is zero. Thus φ is a G - type control function.

Definition 2.3: Let (X, d) be a cone metric space with normal cone P and normal constant K. suppose $S = \{d(x,y) \,/\, x,y \in X\}$ is a totally ordered subset of P. Then (X,d) is called a S-cone metric space.

Theorem 2.4: Let (X, d) be a complete S-cone metric space and P a normal cone with

normal constant K. Suppose that the mappings $f, g : X \to X$ satisfy.

$\min\{d(fx,gy), \max\{d(x,fx), d(y,gy)\}\} \le \Theta d(x,y)$ …. (2.4.1) for some $0 < \Theta < 1$ and for every $x, y \in X$. Then $f = g$ and f has a fixed point in X.

Proof : Put $x = y$ in (2.4.1). We have

min $\{d(fx,gx), max \{d(x,fx),d(x,gx)\} \} \leq \theta d(x,x)$

$\Rightarrow$ min $\{d(fx,gx), d(x,fx)\} \leq 0$ and min $\{d(fx,gx), d(x,gx)\} \leq 0$

$\Rightarrow d(fx,gx) = 0$

$\Rightarrow fx = gx \; \forall x$

$\Rightarrow f = g$

Hence the given condition (2.4.1) reduces to

$min \{ d(fx, fy), max \{ d(x, fx), d(y, fy) \} \} \leq \theta d(x, y)$ ... (2.4.2)

Let $x_0 \in X$

Write $x_1 = f(x_0)$, $x_2 = f(x_1)$, …, $x_n = f(x_{n-1})$, n =1,2……

Let $x = x_n$ and $y = x_{n+1}$

Then, from (2.4.2) , we have

$$min \{d(fx_n, fx_{n+1}), max \{ d(x_n, fx_n), d(x_{n+1}, fx_{n+1}) \} \} \leq \Theta d(x_n, x_{n+1})$$

$\Rightarrow$ min $\{ d(x_{n+1}, x_{n+2}), \max \{ d(x_n, x_{n+1}), d(x_{n+1}, x_{n+2})\}\} \leq \Theta d(x_n, x_{n+1})$

$\Rightarrow d(x_{n+1}, x_{n+2}) \leq \Theta d(x_n, x_{n+1})$

Hence $d(x_{n+1}, x_{n+2}) \leq \theta^n d(x_0, x_1)$

Since $0 < \theta < 1$, follows that $\{x_n\}$ is a Cauchy sequence.

Suppose $x_n \rightarrow x$ and put y = x_n

Then from (2.4.2) we have

$min\{d(fx, fx_n), \max \{d(x, fx), d(x_n, fx_n)\}\} \leq \theta d(x, x_n)$

$\Rightarrow min \{d(fx,x_{n+1}), max \{d(x, fx), d(x_n, x_{n+1})\}\} \leq \theta d(x, x_n)$

On letting $n \rightarrow \infty$, we get

$$min \{ d(fx, x), max \{d(x, fx), d(x, x) \} \} \leq 0 \Rightarrow d(x, fx) = 0 \Rightarrow x = fx$$

Therefore f has a fixed point in X.

NOTE : If we take f as the identity function, clearly (2.4.2) is satisfied and every point of X is a fixed point. Hence in theorem 2.4 we cannot conclude uniqueness of the fixed point.

We give the following example in supporting theorem 2.4

Example 2.5: Let X = [0, $\frac{1}{4}$], E = $R \times R$ and

$P = \{(x, y) \in E \,/\, x \geq 0, y \geq 0\}$. Then P is a normal cone with normal constant 1.

$$Defined: X \times X \rightarrow P by d\,(x, y) = (|y - x|), \frac{1}{2}|y - x|)\; and$$

$$f: X \rightarrow X by f\,(x) = x^2. Then$$

$$\Rightarrow \min\{(\,|\,y^2 - x^2\,|, \frac{1}{2}\,|\,y^2 - x^2\,|), \max\{(\,|\,x^2 - x\,|, \frac{1}{2}\,|\,x^2 - x\,|\,), (|\,y^2 - y\,|, \frac{1}{2}\,|\,y^2 - y\,|\,\}\}$$

$$\leq \frac{1}{2}\,(|\,y - x\,|, \frac{1}{2}\,|\,y - x\,|\,)$$

so that f satisfies condition (1.13.2) with θ = $\frac{1}{2}$

Also 0 is a fixed point of f.

Example 2.6: Let (X,d) be a S – cone metric space and let S,T be self maps on X.
Define $\varphi(x, y) = min$ {d(Sx,Tx), d(Sy,Ty), d(Sx,Ty), d(Sy,Tx)}∀ x, y∈ X then φ is a G-type control function.

Theorem 2.7 : Let (X,d) be a cone metric space and let T,S : X → X be two mappings for which there exist a constant $\mu \in (0, 1)$ and a G – type control function φ such that

d(Tx,Ty) ≤ μd(Sx,Sy) + φ {d(Sx,Tx), d(Sy,Ty), d(Sx,Ty), d(Sy,Tx)} … (2.7.1)

∀ x, y∈ X. If the range of S contains the range of T and S(X) is a complete subspace of X, then T and S have a unique coincidence point in X. Moreover, if T and S are weakly compatible, then T and S have a unique common fixed point in X. In both cases, for any x_0∈ X, the iteration{Sx_n} defined by Sx_{n+1} = Tx_n converges to unique common fixed point(coincidence point) x* of T and S.

Proof: Let x_0 be an arbitrary point in X. Since T(X) ⊂ S(X), we can choose a point x_1 in X such that Tx_0 = Sx_1. Continuing in this way, for x_n in X, we can find x_{n+1} ∈X such that Sx_{n+1} = T x_n , n = 0,1,2 … (2.7.2)

If $x = x_{n-1}$, $y = x_n$ are two successive terms of the sequence defined by(2.7.2), then by (2.7.1) we have

$d(Tx_{n-1},Tx_n) \leq \mu d(Sx_{n-1},Sx_n) + \varphi$ {$d(Sx_{n-1},Tx_{n-1})$, $d(Sx_n,Tx_n)$, $d(Sx_{n-1},Tx_n)$,

$d(Sx_n,Tx_{n-1})$}

$\Rightarrow d(Tx_{n-1},Tx_n) \leq \mu d(Tx_{n-2},Tx_{n-1}) + \varphi$ {$d(Tx_{n-2},Tx_{n-1})$, $d(Tx_{n-1},Tx_n)$, $d(Tx_{n-2},Tx_n)$,

$d(Tx_{n-1},Tx_{n-1})$}

In view of (2.7.2) since $d(Tx_{n-1},Tx_{n-1}) = 0$, the above equation reduces to

$d(Tx_{n-1},Tx_n) \leq \mu d(Tx_{n-2},Tx_{n-1})$

$d(Tx_{n-1},Tx_n) \leq \mu^2 d(Tx_{n-2},Tx_{n-1})$

Hence in general we have

$d(Tx_{n-1},Tx_n) \leq \mu^{n-1} d(Tx_0,Tx_1)$

Now for $p \geq 1$, we get

$d(Tx_{n+p},Tx_n) \leq d(Tx_{n+P},Tx_{n+P-1}) + d(Tx_{n+P-1},Tx_{n+P-2}) + \ldots + d(Tx_{n+1},Tx_n)$

$\leq \mu^{n+P-1} d(Tx_0,Tx_1) + \mu^{n+P-2} d(Tx_0,Tx_1) + \ldots + \mu^n d(Tx_0,Tx_1)$

$= \mu^n(\mu^{P-1} + \mu^{P-2} + \ldots + 1) d(Tx_0,Tx_1)$

$= \frac{\mu^n(1-\mu^P)}{1-\mu} d(Tx_0,Tx_1)$

$\leq \frac{\mu^n}{1-\mu} d(Tx_0,Tx_1)$

Let now $0 \ll \varepsilon$ be given. Choose $\lambda > 0$ such that $\varepsilon + N_\lambda(0) \subset$ int P, where

$N_\lambda(0) = \{y \in E : \| y \| < \lambda\}$. Also choose a natural number N_1 such that

$\frac{\mu^n}{1-\mu} d(Tx_0,Tx_1) \in N_\lambda(0)\ \forall\ n \geq N_1$

Then $\frac{\mu^n}{1-\mu} d(Tx_0,Tx_1) \ll \varepsilon \forall\ n \geq N_1$

Hence $d(Tx_{n+p},Tx_n) \leq \frac{\mu^n}{1-\mu} d(Tx_0,Tx_1) \ll \varepsilon \forall\ n \geq N_1$which shows that $\{Tx_n\}$ is a Cauchy sequence and hence $\{Sx_n\}$ is also Cauchy.

Since S(X) is complete, there exists x^* in S(X) such that$\log_{n\to\infty} Tx_n = \log_{n\to\infty} Sx_n = x^*$

… (2.7.4)

We can findp $\in$ X such that Sp = x^* (since $x^* \in$ S(X))

Now we show that $\{Tx_n\} \to$ Tp

We have, by (2.7.1)

$d(Tx_n,Tp) \leq \mu d(Sx_n,Sp) + \varphi\ \{d(Sx_n,Tx_n), d(Sp,Tp), d(Sx_n,Tp), d(Sp,Tx_n)\}$

On letting n → ∞, we get that

$\lim_{n\to\infty} d(Tx_n,Tp) \leq \lim_{n\to\infty} \mu d(Sx_n,Sp) + \varphi\ \{d(Sx_n,Tx_n), d(Sp,Tp), d(Sx_n,Tp), d(Sp,Tx_n)\}$

$= \lim_{n\to\infty} \mu d(Sx_n,Sp) + \varphi\ \{\lim_{n\to\infty} d(Sx_n,Tx_n), d(Sp,Tp), \lim_{n\to\infty} d(Sx_n,Tp),$

$\lim_{n\to\infty} d(Sp,Tx_n)\}$

$= \mu d(x^*,x^*) + \varphi\ \{d(x^*,x^*), d(x^*,Tp), d(x^*,Tp), d(x^*,x^*)\}$

$= 0$

So that $\lim_{n\to\infty} d(Tx_n,Tp) \leq 0$

Hence $\lim_{n\to\infty} d(Tx_n,Tp) = 0$

Thus $Tx_n \to Tp$ …(2.7.5)

By (2.7.4) and (2.7.5) follows that $Tp = Sp = x^*$

⇒pis a coincidence point of T and S or x^* is a point of coincidence of T and S

Now we prove that x^*, point of coincidence of T and S, is unique.

$Tx = Sx = y^*$ and $Sp = Tp = x^*$ be two points of coincidence of T and S.

Then we show that $x^* = y^*$

We have from

$d(Tx,Tp) \leq \mu d(Sx,Sp) + \varphi\ \{d(Sx,Tx), d(Sp,Tp), d(Sx,Tp), d(Sp,Tx)\}$

$= \mu d(Tx,Sp) + \varphi\ \{0, 0, d(Sx,Tp), d(Sp,Tx)\}$

$\Rightarrow d(Tx,Tp) \leq \mu d(Tx,Sp) + 0$

$= \mu\, d(Tx,Tp)$

$\Rightarrow d(Tx,Tp) = 0$

$\Rightarrow Tx = Tp$

$\Rightarrow x^* = y^*$

Thus x^* is the unique point of coincidence of T and S

Now suppose T and S are weakly compatible.

Then, by Proposition 1.9, x^* is the unique point of coincidence of T and S

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